Ch5 : Vector Space

Let $S = \{v_1, v_2, v_3 \dots\}$ in $\mathbb{R}^n$ / P_n / M_{MN} (V)

→ Span :

If $K_1v_1 + K_2v_2 + K_3v_3 + \dots = (a, b, c \dots)$

↳ has solution ⇒ S span V

↳ has no solution ⇒ S not span V

→ Linear Independence

If $K_1v_1 + K_2v_2 + K_3v_3 = 0$

↳ has unique solution. $(K_1 = K_2 = K_3 = 0) \rightarrow$ S LI

↳ has many solution → S LD.

. If $\det(A) = 0$ ⇒ many sol → L.D

. If $\det(A) \neq 0$ ⇒ unique sol → L.I.

→ Subspace

S is a subspace of V if:

1. S is non empty $S \neq \{\emptyset\} \Rightarrow 0 \in S$
2. Let $\vec{u}, \vec{v} \in S$: $\vec{u} + \vec{v} \in S$
3. Let a scalar : $a\vec{u} \in S$.

→ Basis

S is basis for V if

. S is linearly idependent

and . S Spans V

→ dimension

∴ nb of vectors in a basis of V

∴ nb of parameters in V

Rq/ If $\dim(V) = n$ and element $S = n$
the S is a basis of V if
S is L.I or Span$V.

→ Direct Sum of vector space

U and W subspaces of V

$V = U \oplus W$ (V direct sum of U and W)

if; 1) $V = U + W$ $\{ \vec{u} + \vec{w} \;/\; \vec{u} \in U \;;\; \vec{w} \in W \}$

2) $U \cap W = 0$